Chapter IX : SKIZ and Watershed

Distance function

Euclidean and Geodesic SKIZ

Watersheds

Definition and properties

Algorithms

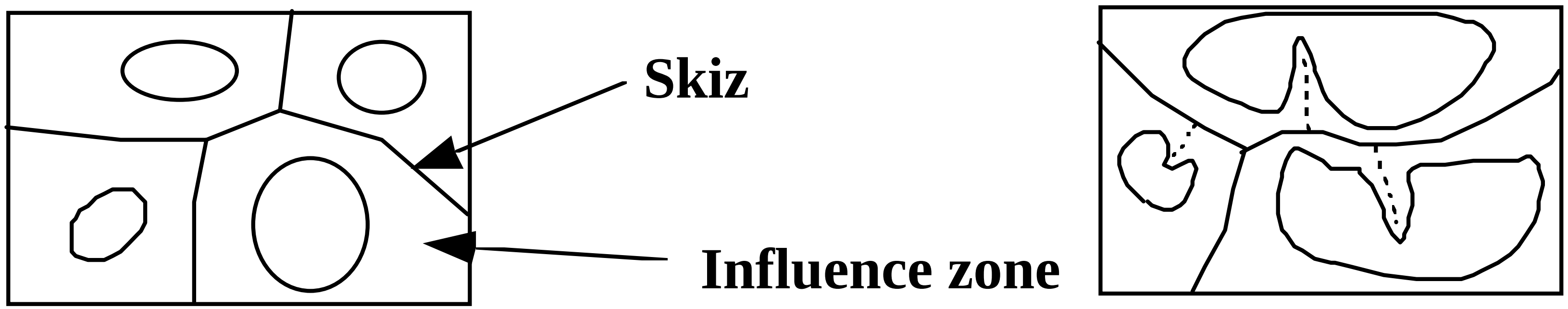
SKIZ, or Skeleton by Influence Zones

***Definition** (C.Lantuejoul)* Consider a compact set X of R^2.

- The ***zone of influence*** of a component X_i of X is the set of points of the plane that are closer to X_i than to any other component.
- The **SKIZ** is then defined as the boundary of all zones of influence.

Algorithm

- In the digital case, the SKIZ is constructed in two steps:
  1) Thinning of the background (with L in the hexagonal case)
  2) Pruning of the thinned transform (with E in the hexagonal case)



Geodesic SKIZ

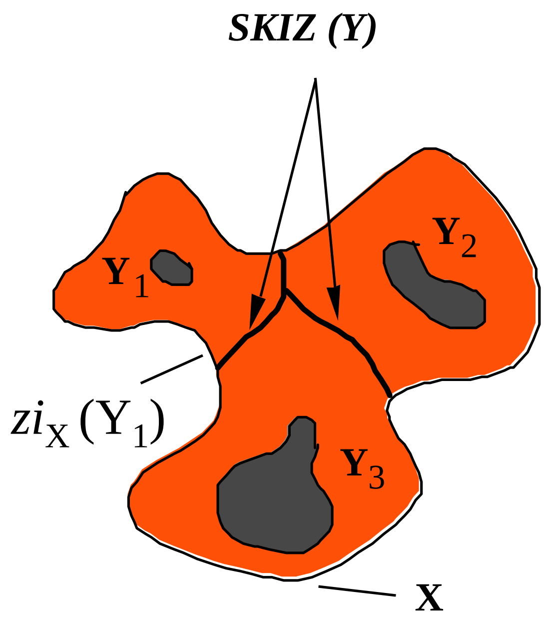
Let $Y = \cup\{ Y_i , i\in I \}$ be a set of the Euclidean plane made of of I compact connected components, and included in a compact set X.

- The ***geodesic zone of influence*** of a component Y_i in X, is formed by all points of X whose geodesic distance to Y_i is smaller than to any other component of Y

$$zi\,(Y_i \setminus X) = \{a\in X\ , \forall k \neq i,\ d_X(a, Y_j) \leq d_X(a,Y_k)\}$$

where the geodesic distance from point a to set Y is the inf of the geodesic distances from a to all points of Y.

- The ***geodesic SKIZ*** is then the boundaries of all geodesic zones of influence.



An example of geodesic SKIZ

Properties of the Skiz

In the following, we assume that set $X \subseteq R^2$ is finite union of ***I*** disjoint compact connected components K_i . Any image of [0,1] under a bi-continuous bijection is called a *simple arc*.The two properties which follow are due to *C.Lantuejoul*.

- ***Fineness***

The set ***skiz*(X)** is a locally finite union of simple arcs, which generate exclusively loops (including possible points at the infinity).

- ***Continuity***

Let $X_n = \{ \cup K_{i,n} \}$ be a sequence of the above type, where each compact set $K_{i,n}$ converges towards compact set K_i , itself disjoint from the other limits K_j , $j \neq i$, $j \in$ ***I*** *i.e.*

$$\mathbf{X_n = \cup K_{i,n} \rightarrow X = \cup K_i}$$

then **Skiz (X_n) → Skiz (X) .**

The Two Problems of Segmentation (I)

- When one wants to segment a set, the first question which arises is :

  " ***into how many pieces ?*** "

  (in case of figure 1, 6 or 7 particles?)

- One can decide and indicate, manually, the supposed locations of the centres.

- Alternatively, one can trust in a marking technique. However, the results risk to vary with the method (here, between 6 and 7).

- In all cases, this first step is a **choice**.

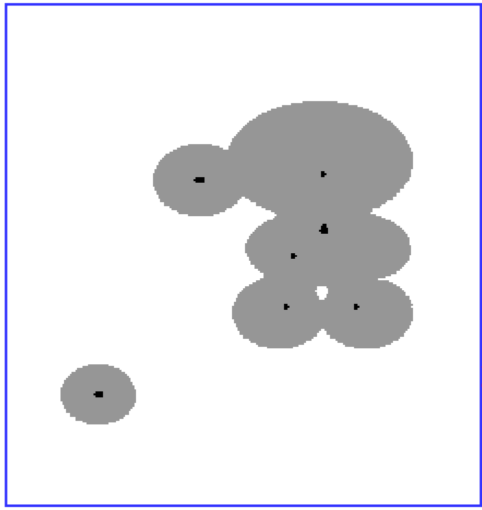
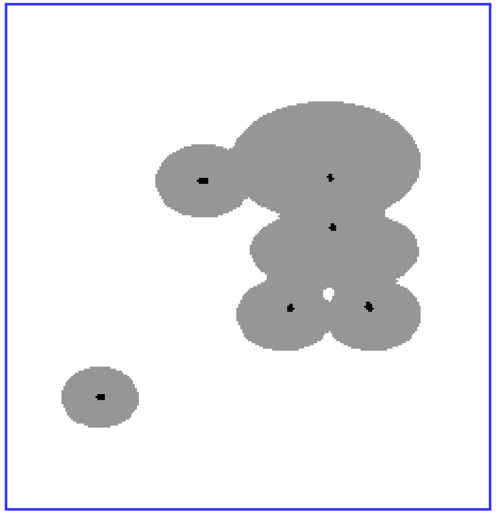


Figure 1 :

A set and its ultimate erosion



Ultimate erosion after filtering of the set.

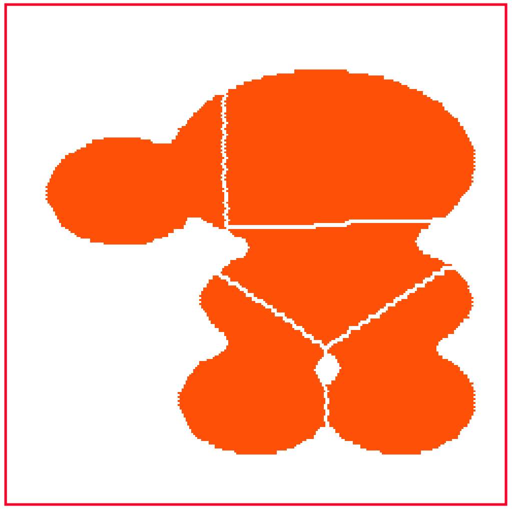
The Two Problems of Segmentation (II)

- Given a certain choice of markers, (here, the conditional bisector) the segmentation lines may be **optimised :**

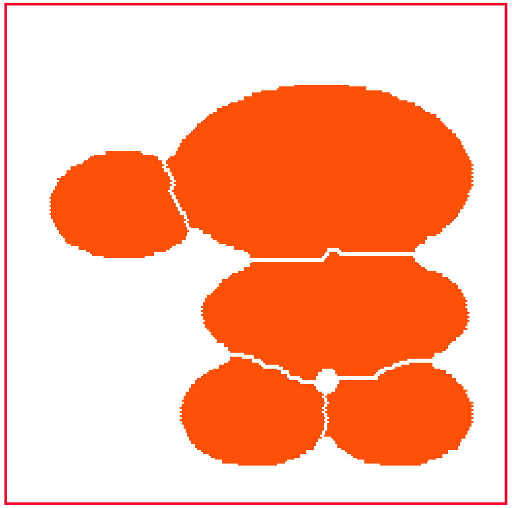
- A coarse expression is obtained by taking the exoskeleton of the markers (the shape of the set is then just ignored)

- One can partly take this shape into account by dilating each marker by a disc equal to the number of steps necessary to reach the ultimate marker

- Finest procedure: calculate the geodesic skiz of eroded n° i inside eroded n° i-1, as i varies from the ultimate erosion to zero, and take the union of these skiz's.



Exoskeleton of the markers.



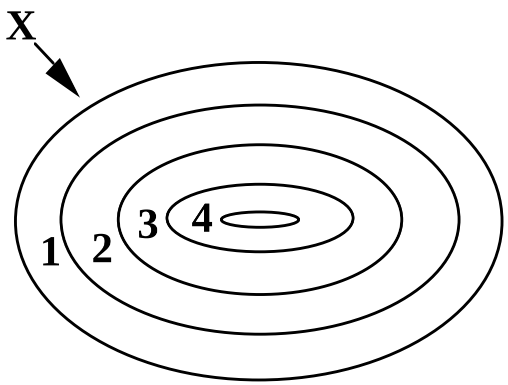
Successive geodesic skiz's.

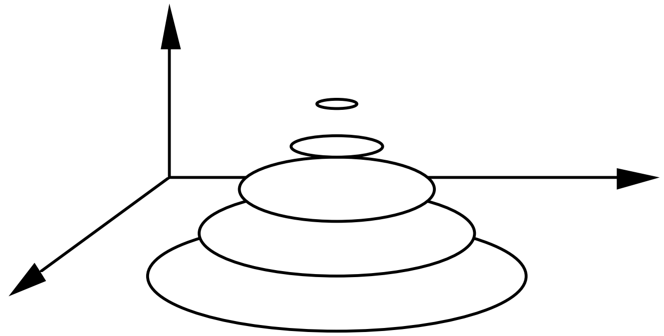
Distance function (I)

Definition :

The distance function is an intermediate step between sets and functions.

- When a distance has been defined on E, it is possible to associate, with each set X, the subset X_λ composed of all those points of X whose distance to the boundary is larger than λ.





- As λ increases, the subsets X_λ are included within each other (and parallel in the Euclidean case). They can be considered as the horizontal thresholds of a function whose grey level is λ at point x if x is at distance λ to the boundary. This function is called **Distance Function**.

Distance Function (II)

Properties

- Since the distance is characterised by the disks δ_λ of size λ, the subsets X_λ are nothing but the erosions of X by these disks. More precisely:

  i **)** $\lambda \geq \mu \Rightarrow \delta_\lambda \geq \delta_\mu$

  ii **)** $\delta_\lambda \delta_\mu \leq \delta_{\lambda+\mu} \qquad \lambda, \mu \geq 0$ **(triangular inequality)**

  iii **)** $\wedge\{\delta_\lambda \ , \lambda \geq 0\} = \mathbf{Id.}$ **(Identity operator)**

  iv **)** $\mathbf{x} \subseteq \delta\,(\mathbf{y}) \Leftrightarrow \mathbf{y} \subseteq \delta\,(\mathbf{x})$ **(symmetry)**

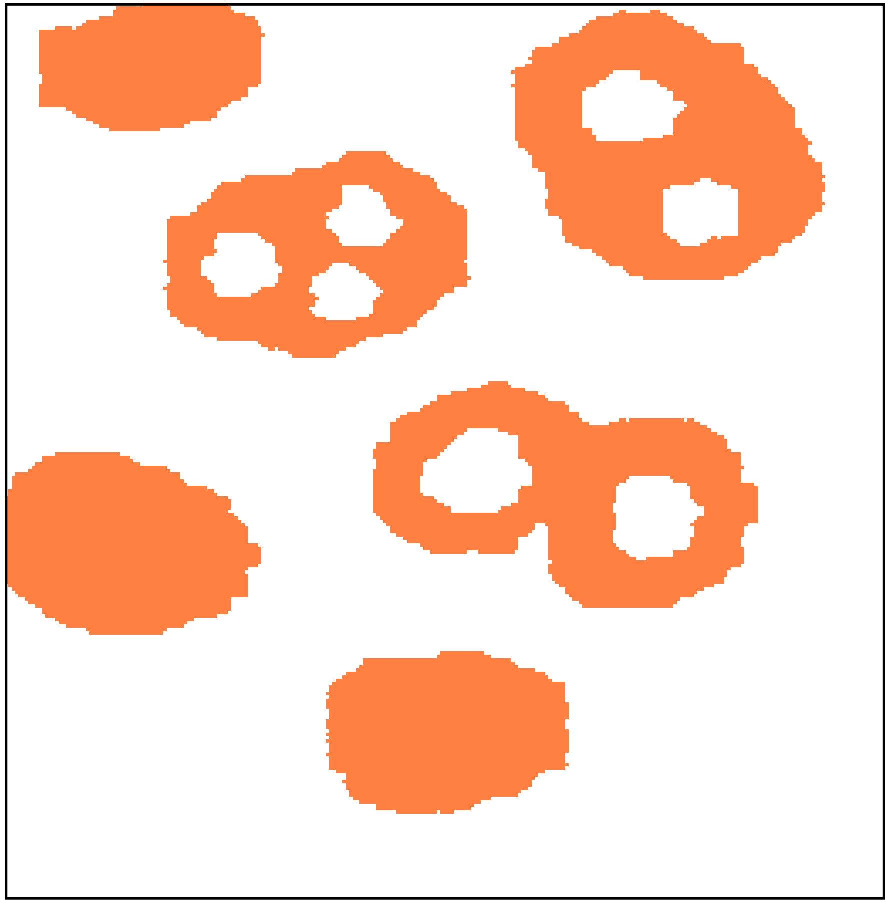
- Conversely, each family of dilations which fulfils these four relations defines a ***distance*** **d** which is characterised by the relation

$$\mathbf{d(x,y) = Inf}\ \{\lambda : \mathbf{x} \in \delta_\lambda\,(\mathbf{y})\ ;\ \mathbf{y} \in \delta_\lambda\,(\mathbf{x})\ \}$$

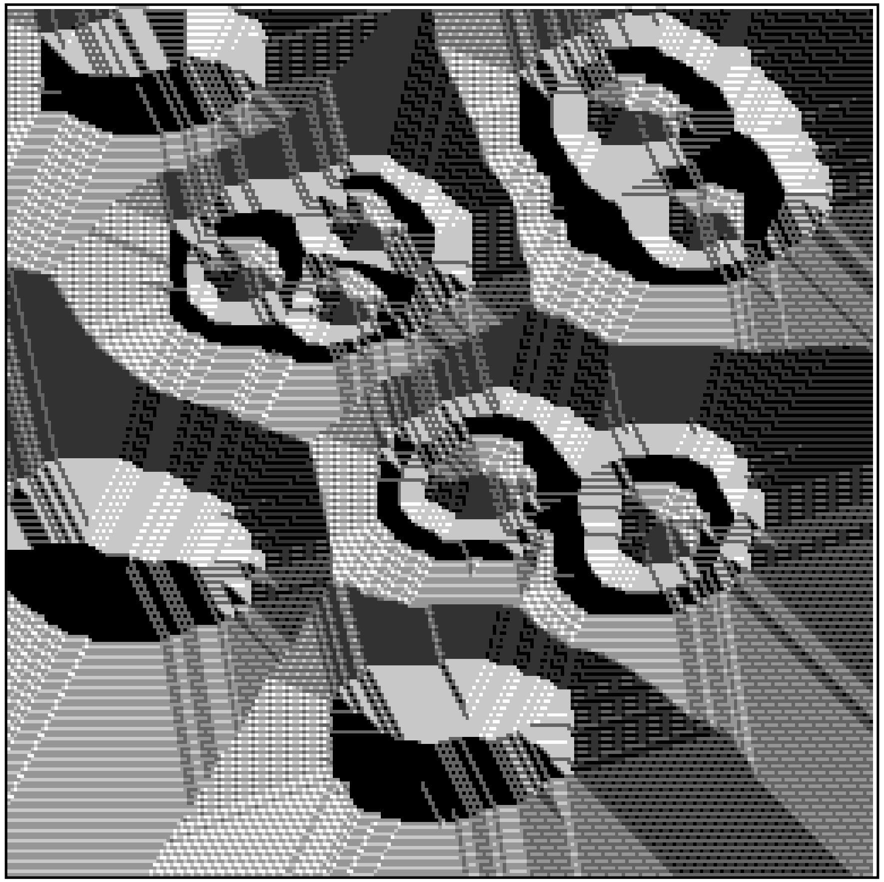
Then δ_λ (y) is the disc of centre y and radius λ *(J. Serra)* .

Distance Function (III): an Example

Comment : ***Figure b shows the supremum of both distance functions of X and X^c***



Set X



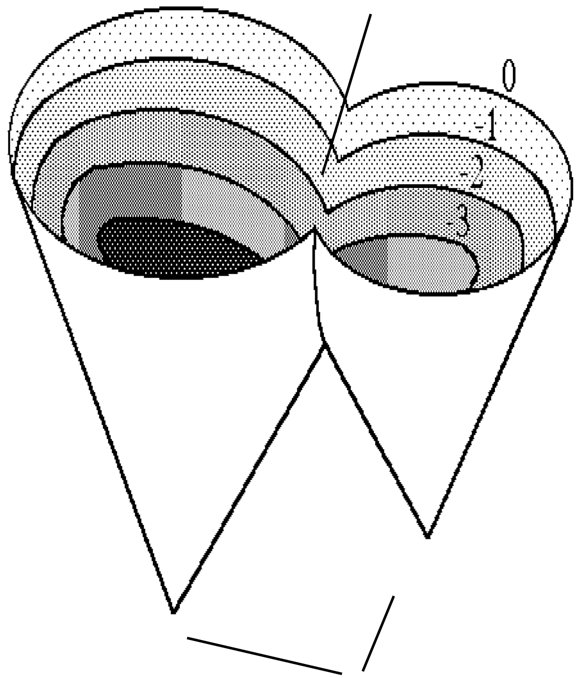
Corresponding Distance Functions

Distance function (IV): another Example

The journey of Men and Women Tingary (Papunya, Australia)

Fine Segmentation and Distance Function

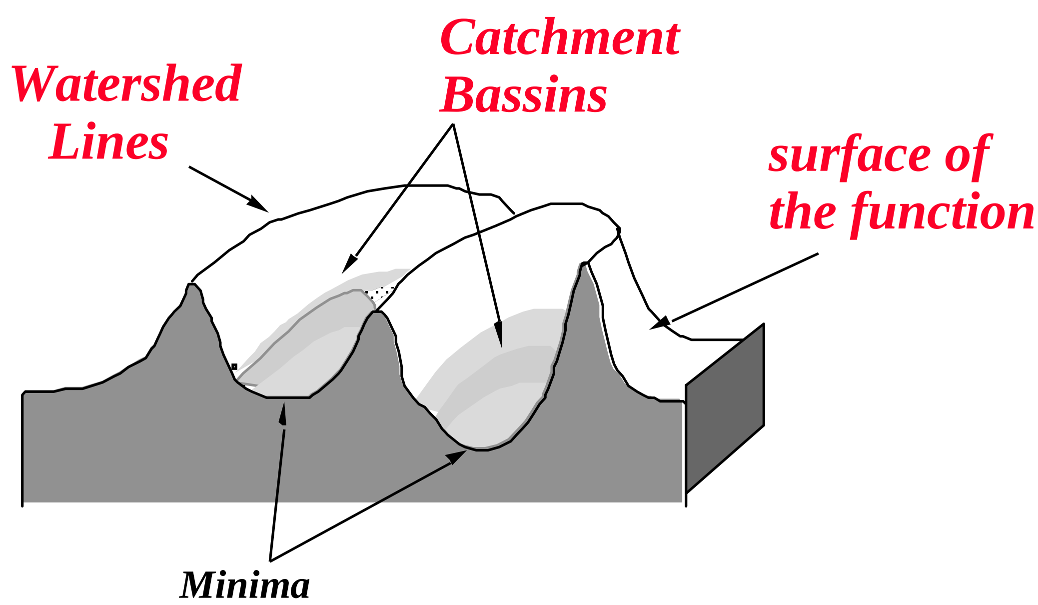
- The successive eroded versions of a set X generate the horizontal sections of its distance function ;
- therefore the finer previous segmentation, by means of geodesic skiz's, comes back to build up the **watershed lines** of this distance function (at least when the les markers are the ultimate erosions).
- By duality, they also appear to be the **valleys lines** on the inverse function.



Watershed Lines for Numerical Functions

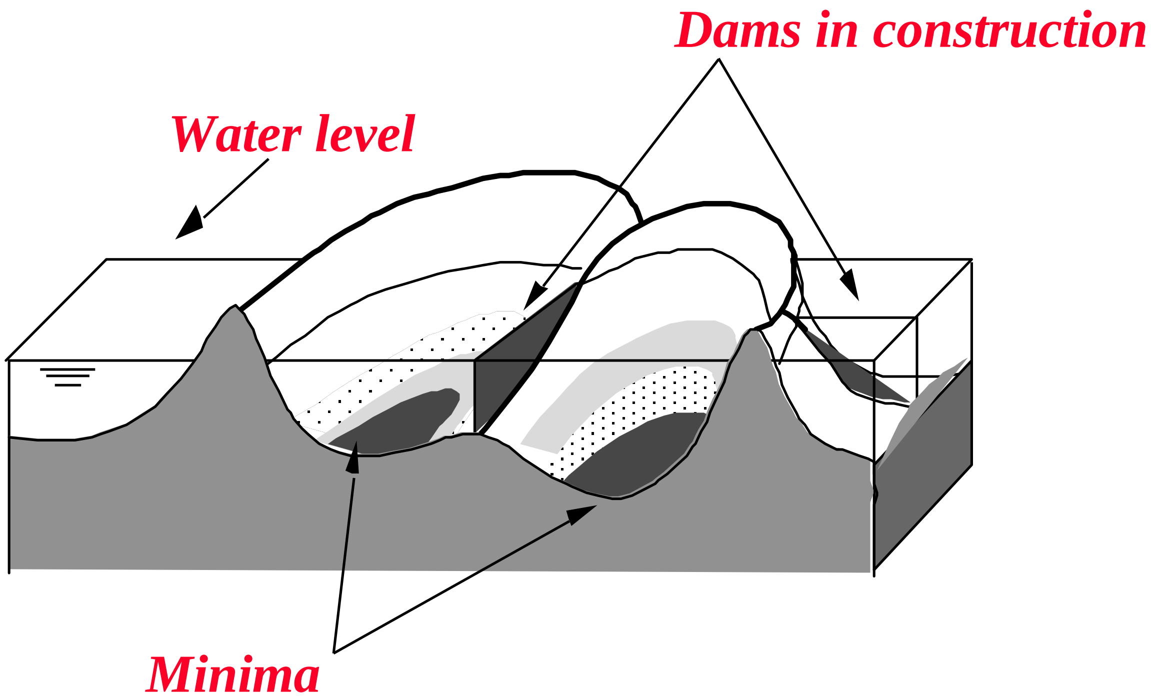
The approach developed for the distance functions applies as well to any **numerical image** (*S.Beucher*), and the analogy between gray levels and altitudes still justifies the terms of **watersheds** and **catchment bassins** .

However, it is less matter of rain water running down to the minima than of water that **springs from the minima** .



N.B. *Beucher-Lantuejoul's algorithm is presented below for the sake of pedagogy. But it is not the unique one, it has been improved by P.Soille and L.Vincent. Another implementation, based on hierarchical queues, due to F.Meyer is more effective.*

Construction of the Watersheds by Flooding (I)



- Suppose that holes are made in each local minimum and that the surface is flooded from these holes. Progressively, the water level will increase.
- In order to prevent the merging of water coming from two different holes, a dam is progressively built at each contact point.
- At the end, the union of all complete dams constitute the **watersheds**.

Construction of the Watersheds by Flooding (II)

Flooding algorithm *(S.Beucher, Ch Lantuejoul)*

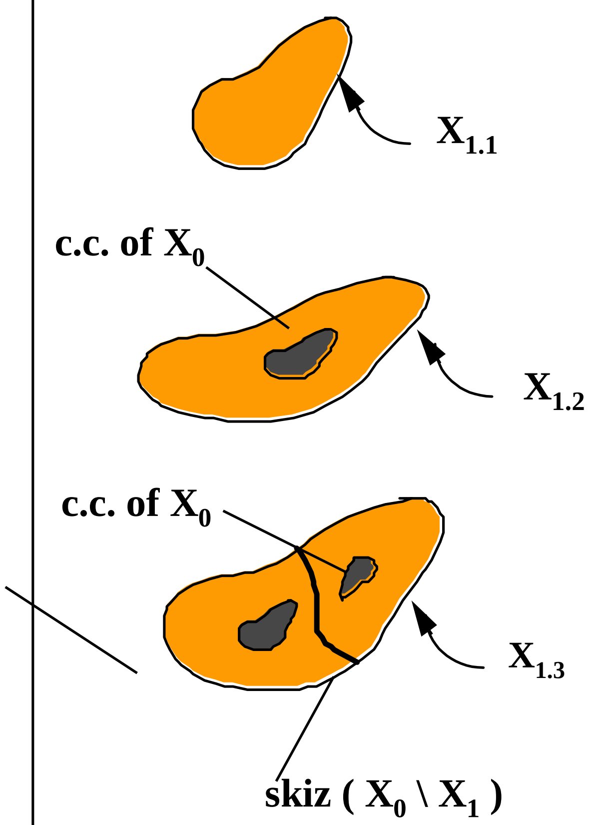
- Let m be the minimum of function f. Put:

  $X_0 = \{ x: f(x) = m\}$,

  $X_k = \{ x: f(x) \leq m+k \}$ with $1 \leq k \leq \max f$

- Denote by Y_1 the geodesic zones of influence of X_0 inside X_1. Distinguish three types of connected components of X_1
  - those, $X_{1,1}$ that do not contain points of X_0 : then they do not belong to Y_1
  - those, $X_{1,2}$ that contain a unique c.c. of X_0 : then they fully belong to Y_1
  - those, $X_{1,3}$ that contain several c.c. of X_0 : Y_1 recovers then $X_{1,3}$ minus the branches of its geodesic skiz.

Evolution of the flooding



Construction of the Watersheds by Flooding (III)

- Since the $X_{1,1}$'s are minima which appear at level 1, we have to incorporate them to the flooding process. Thus we replace

$$X_1 \quad \text{by} \quad Y_1 \cup X_{1,1}$$

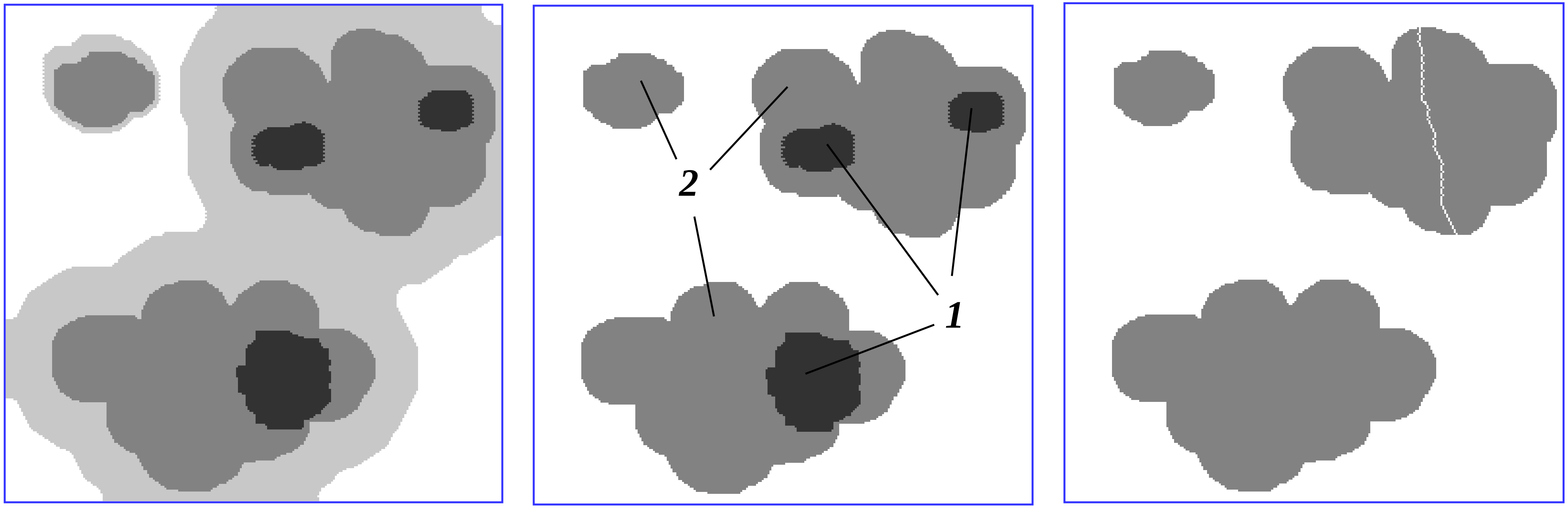
- ...and we iterate. The geodesic zones of influence
  - Y_2 of $Y_1 \cup X_{1,1}$ inside X_2 are calculated ;
  - They provide markers $Y_2 \cup X_{2,1}$; etc...

- The process ends when level k = max f is reached. Then one has :

$$\mathbf{Y_{max\ f} \quad = \quad union\ of\ the\ basins;}$$

$$\mathbf{[\ Y_{max\ f}\ ]^c = \ Watershed\ lines.}$$

An example of Watershed by Flooding (I)

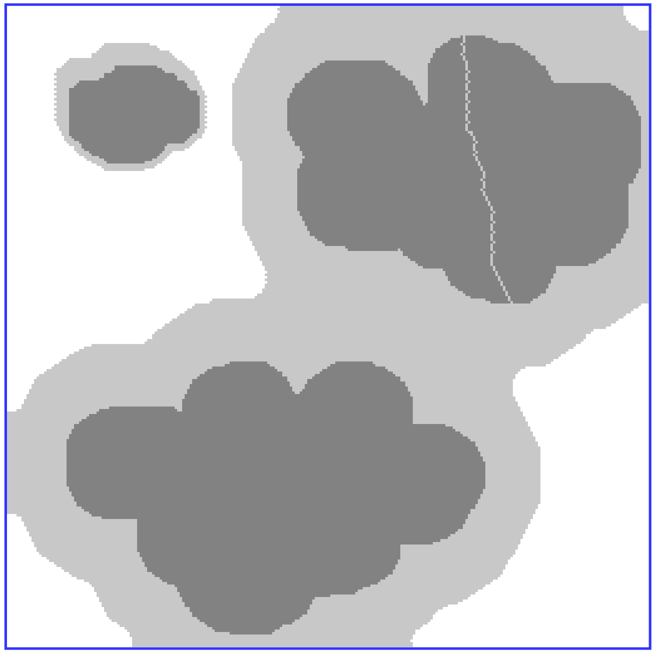


Initial image.

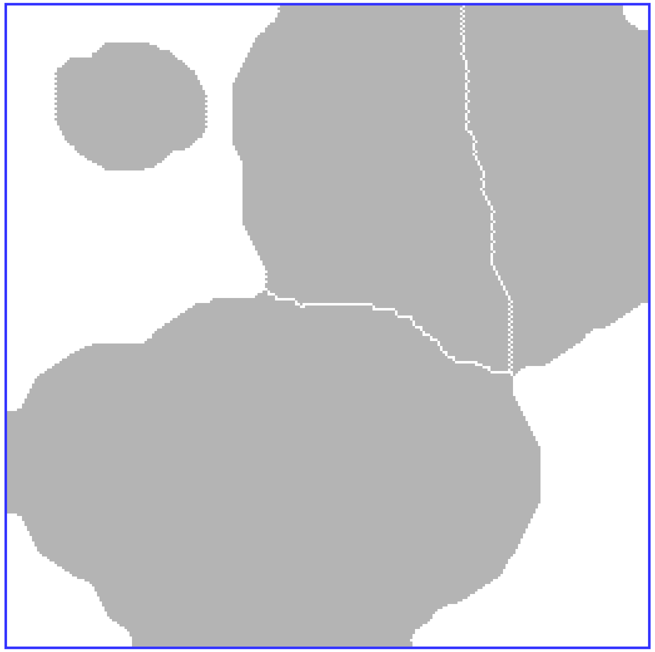
Minima (1), and next level (2).

Geodesic skiz of (1) into (2) (in white lines).

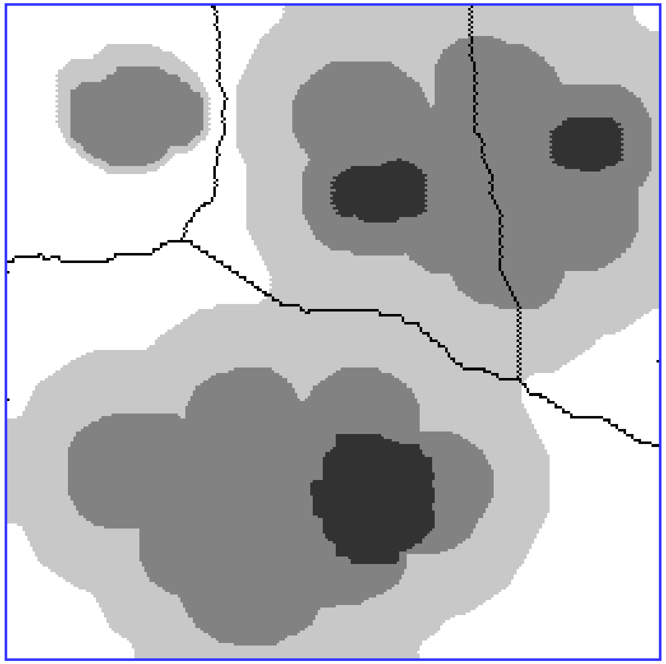
An example of Watershed by Flooding (II)



Level 2, minus the first skiz , and level 3.



Second skiz (note that it prolongates the first one).



Final watershed (The result is significant in spite of the small number of gray levels).

References

On distance function :

- The concept of parallel sets (that is Euclidean spherical dilates) appears in Steiner{STE40} in 1840. But the distance function was introduced by G.Matheron in 1967 for the Euclidean case {MAT67}, and by A.Rosenfeld in 1968 for digital sets {ROS68}. Literature provides a large number of algorithms to compute this function {MEY89a},{VIN90},{SOI91} and {DAN80}. For the dilation based distances, see {SER88, ch2}.

On Watersheds :

- Watershed transformation was originally designed for segmentation of gray tone images by S.Beucher and Ch.Lantuejoul in 1979 {BEU79}. During the eighties, S.Beucher and F.Meyer proposed the concepts of markers and of swamping, and published them in {BEU90} and {MEY90}.The most efficient algorithm for watershed implementation was found by F.Meyer {MEY91} (hierarchical queues). See also {SER98} (hyper-connections) and {ALB97} (enlargements) .