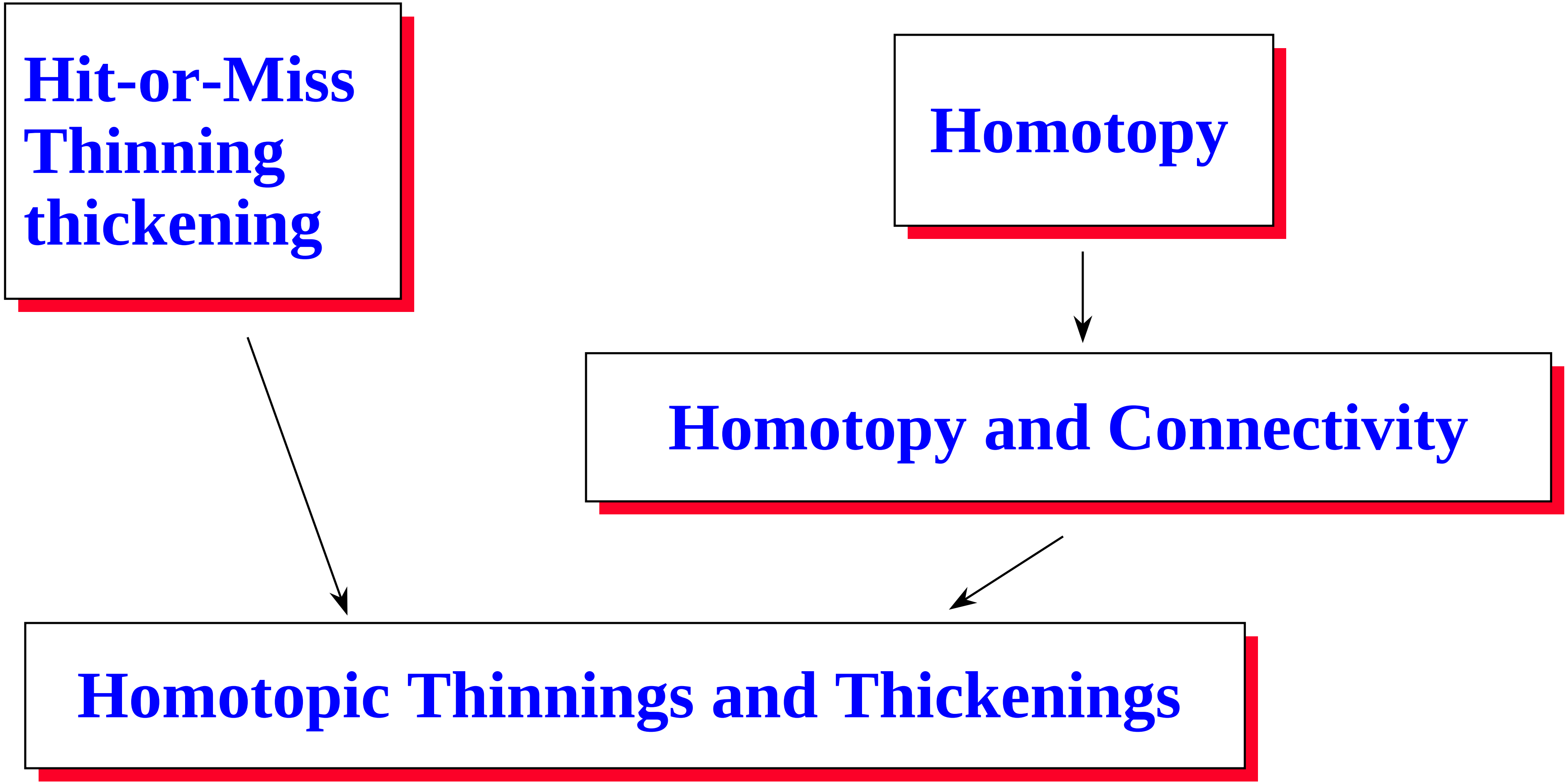
Chapter VIII : Thinnings

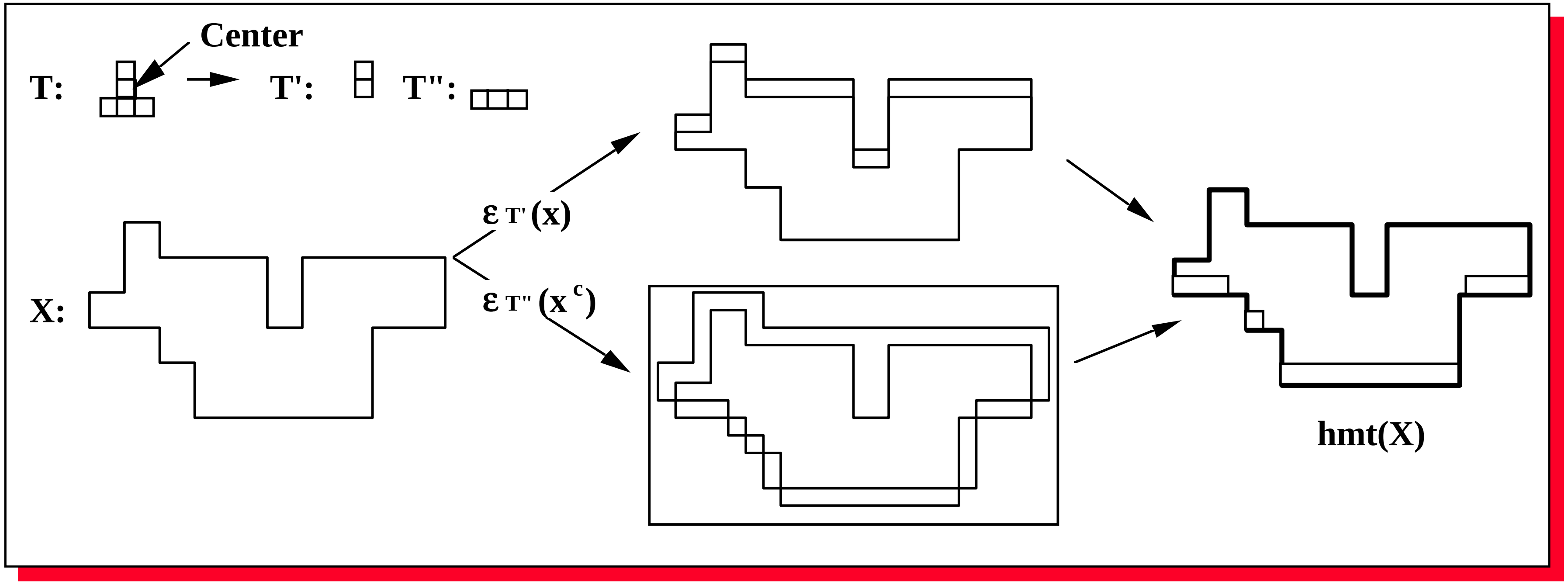


"Hit or Miss" Transformation

Definition (J.Serra)

- The "Hit or Miss" mapping η_T (sect.II-2) generalises both erosion and dilation. It involves a pair of structuring elements T' and T": T=(T',T")

$$\eta_T(X) = \{z: T''(z) \subseteq X^c ;\ T'(z) \subseteq X\} = \varepsilon_{T'}(X) \cap \varepsilon_{T''}(X^c)$$

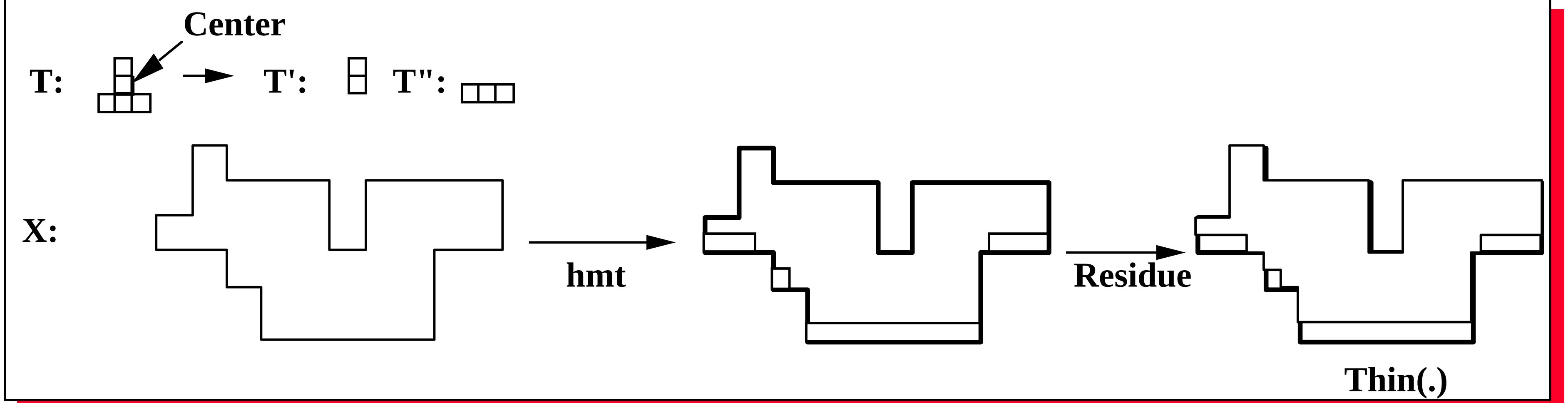


"Thinning" and "Thickening"

***Definition** (A. Rosenfeld, J.Serra)*

- The **thinning** θ_T is the residue between the initial set and its transformation by "hit or miss":

$$\theta_T(X) = X \setminus \eta_T(X) = X \setminus [\varepsilon_{T'}(X) \cap \varepsilon_{T''}(X^c)]$$



- By duality for the complement, the **thickening** ξ_T is defined as:

$$\xi_T(X) = X \cup \eta_T(X) = X \cup [\varepsilon_{T'}(X) \cap \varepsilon_{T''}(X^c)]$$

Properties

***The Representation Theorem** (G.Banon and J.Barrera) :*

- Every translation invariant operator ψ on $\mathcal{P}(E)$ is represented as a union of Hit-or-Miss operators :

$$\psi(X) = \cup \, \eta_{T\,i}(X)$$

for a convenient family $\{T_i', T_i''\}$ depending on the kernel of ψ.

Duality :

- "Thinning" and "Thickening" are dual in the following sense:

$$\theta_{T',T''}(X) = [\, \xi_{T'',T'}(X^c)\,]^c$$

In particular, thinning (resp. thickening) is anti-extensive (resp. extensive) and not trivially reduced to Identity mapping when $0 \in T'$ (resp. $0 \in T''$).

Use :

- In morphology, "thinning" and "thickening" are used to define transformations preserving the homotopy, and i
- n particular to compute homotopic skeletons.

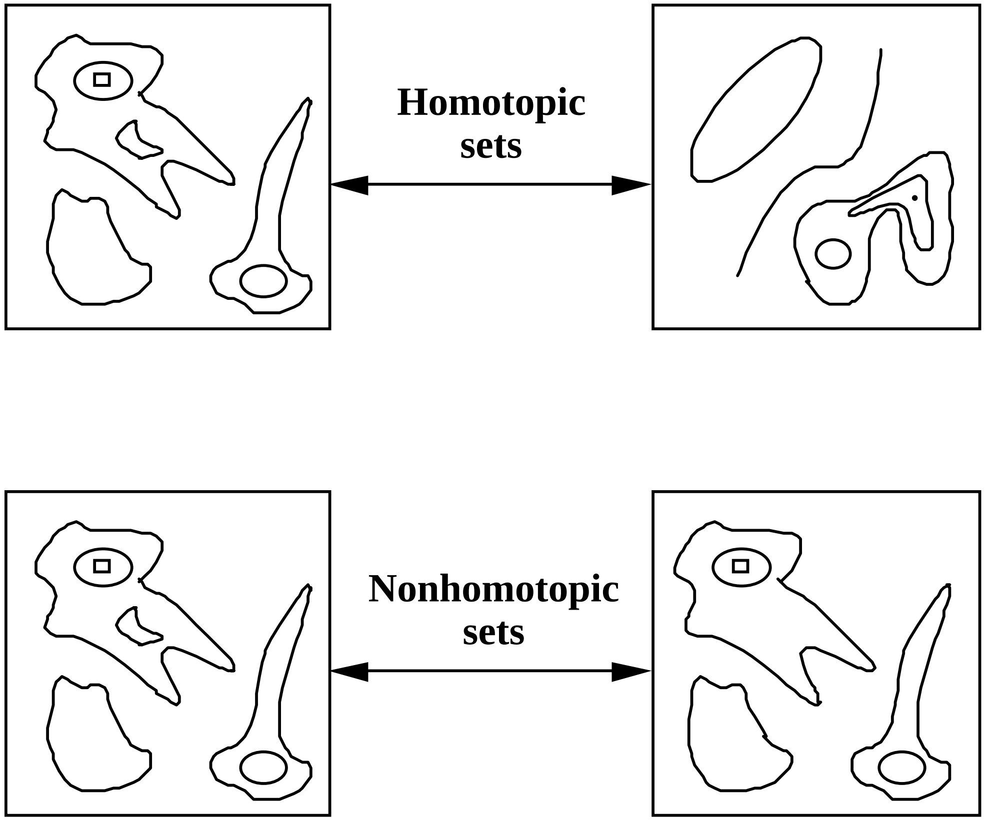
Homotopy of sets

Definition

- Two sets are homotopic if there exists a bi-continuous mapping from one to the other such that:

  -each grain and its transform must contain the same number of holes,

  -and each hole and its transform the same number of grains.

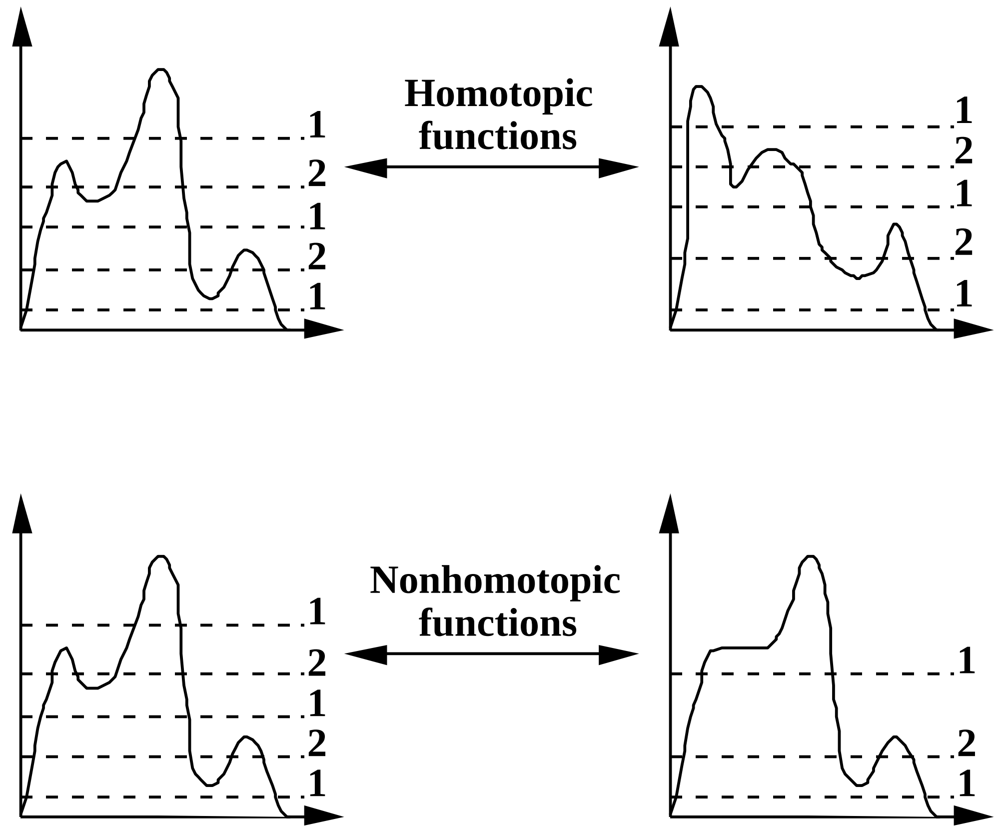


Homotopy for Functions

***Definition** (J.Serra)*

- The notion of homotopy for functions is defined via their cross sections. Two functions are homotopic when one can find a grey levels anamorphosis which makes homotopic the sections of same levels.

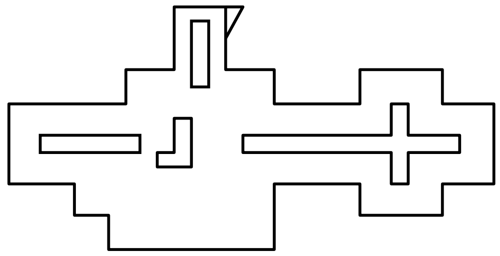
- Intuitively, the homotopy characterises the structures of **minima, maxima** and **saddle points**.



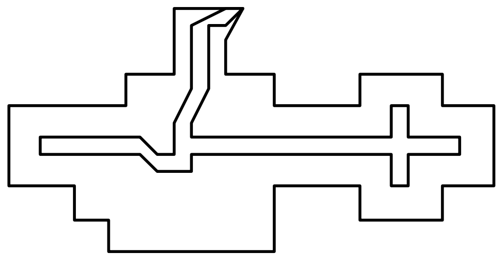
Homotopic Transformations

Definition

- A transformation is homotopic when its input and output have the same homotopy.
- Until now, the only homotopic transformation that has been seen is the skeleton in the *Euclidean* case. In digital, this property disappears.
- In order to preserve digital homotopy, we will now replace skeletons by *thinnings* substitutes.



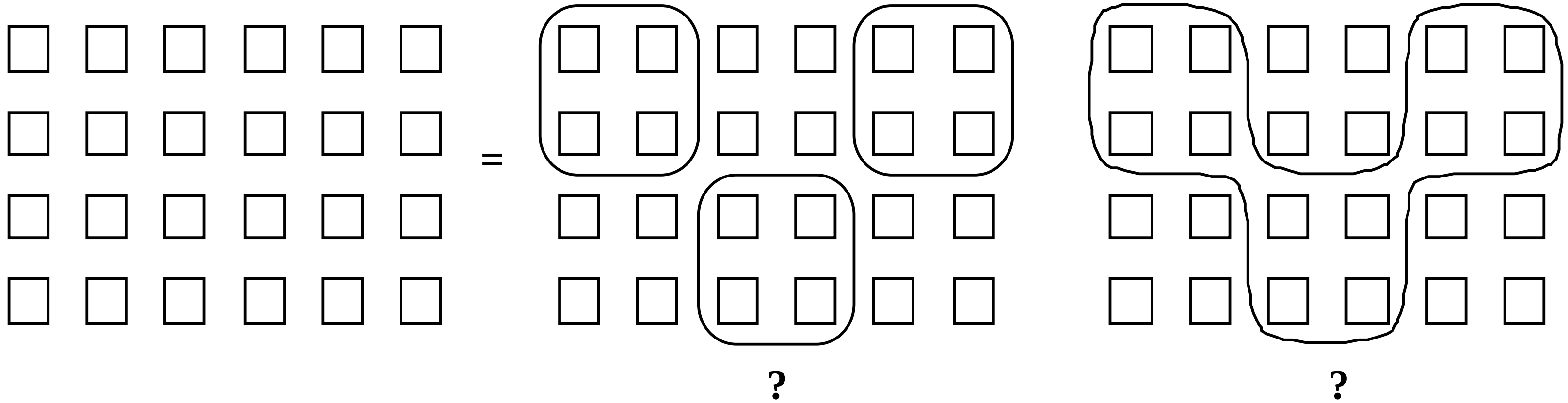
Digital skeleton



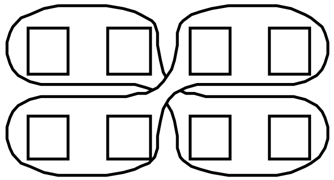
Continuous skeleton

Homotopy and Digital connectivity

- In the digital case, the definition of connectivity and as a consequence of homotopy is not unique. For example, the number of sets in the following figure is not *a priori* obvious:



- To define the connectivity, it is necessary to define connection rules between foreground points and between background points.
- Note: The connection rules should prohibit the cross-connection between foreground and background points:



Connectivities for Square Grid (I)

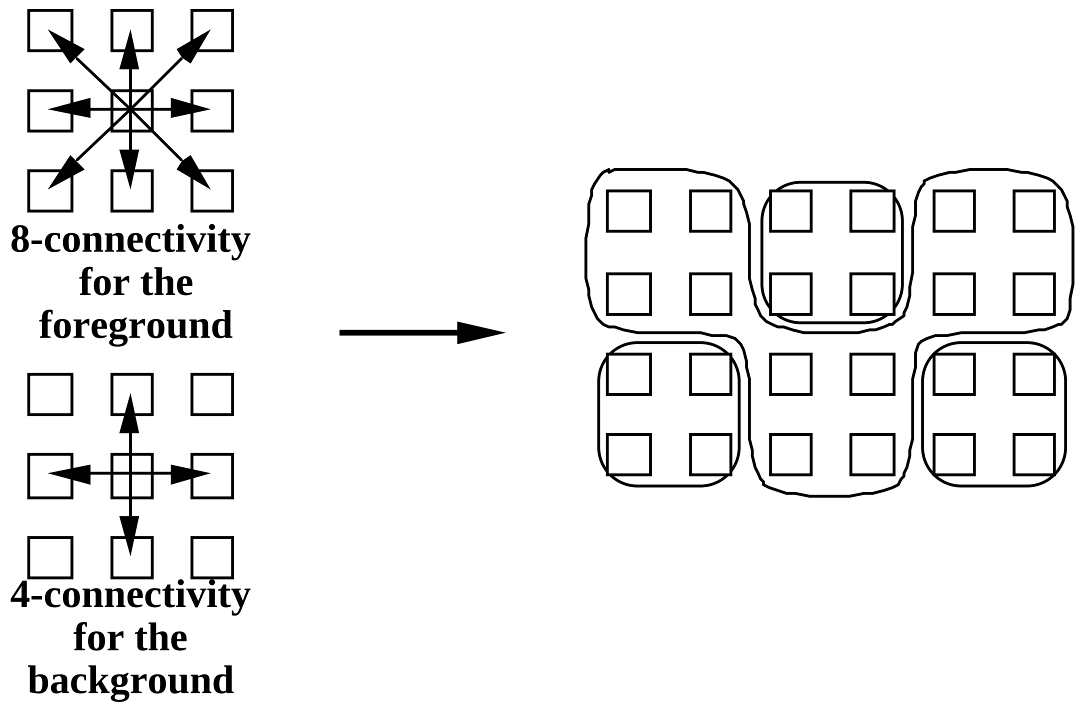
In practice, three arcwise connections are used.

1rst case(A. Rosenfeld)

8-connectivity for the foreground
4-connectivity for the background

Properties

- Translation invariance
- rotation invariance of 90°
- Not self-dual.



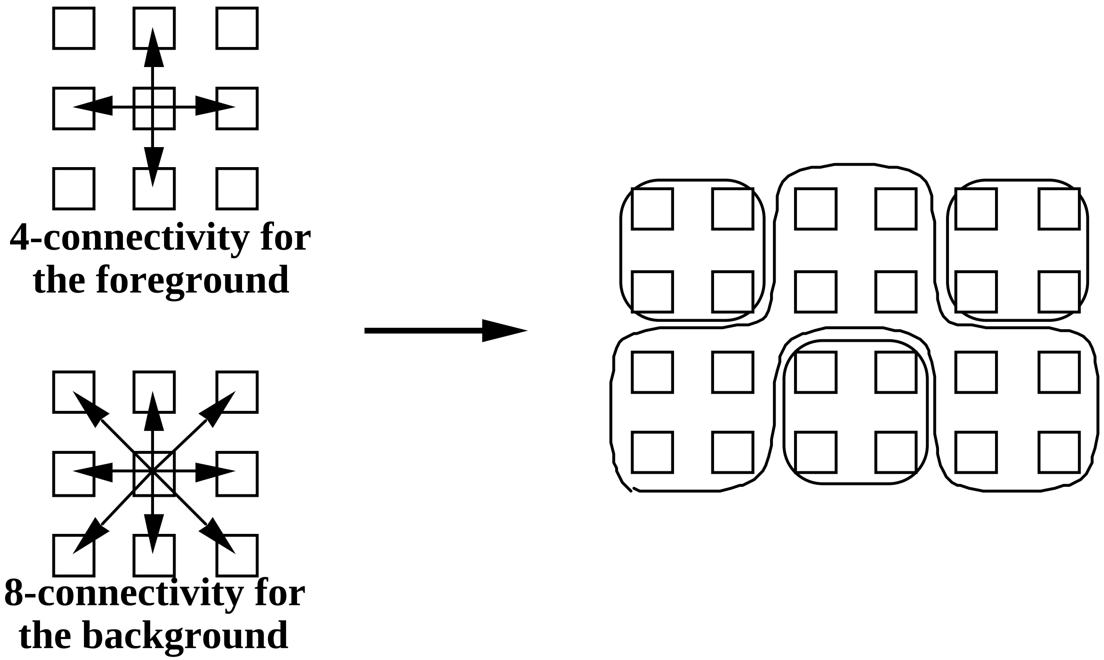
Connectivity for Square Grid (II)

2nd case (A. Rosenfeld)

4-connectivity for the foreground
8-connectivity for the background

Properties

- Translation invariant
- rotation invariance of 90°
- Not self-dual.



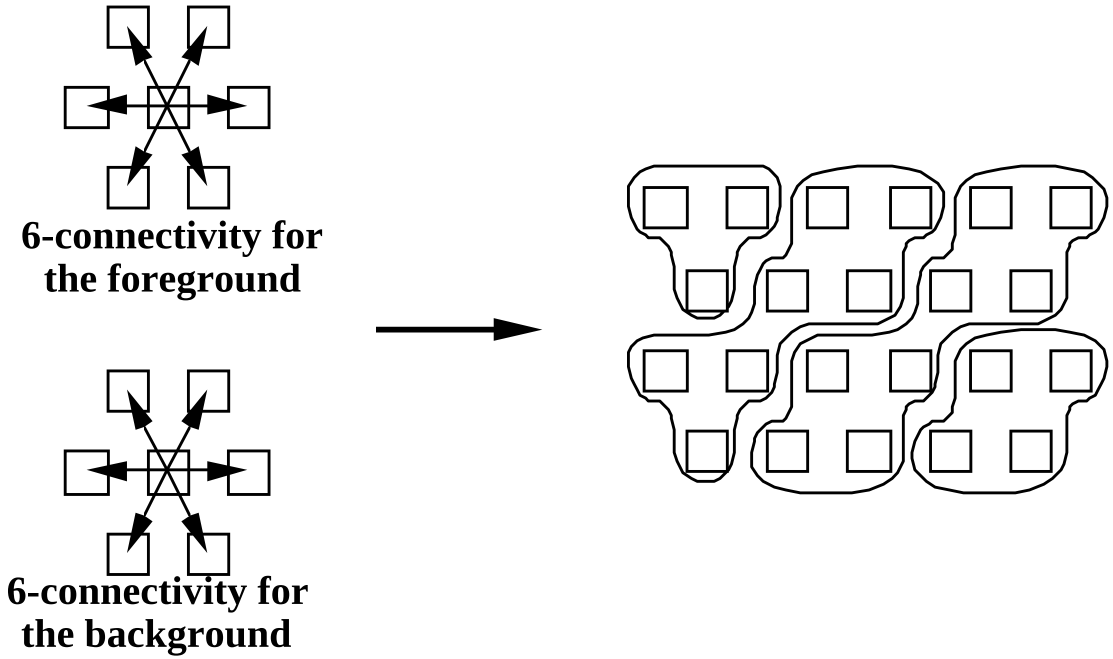
Connectivity for Hexagonal Grid

3rd case (M.J.E. Golay)

6-connectivity for the foreground
6-connectivity for the background

Properties

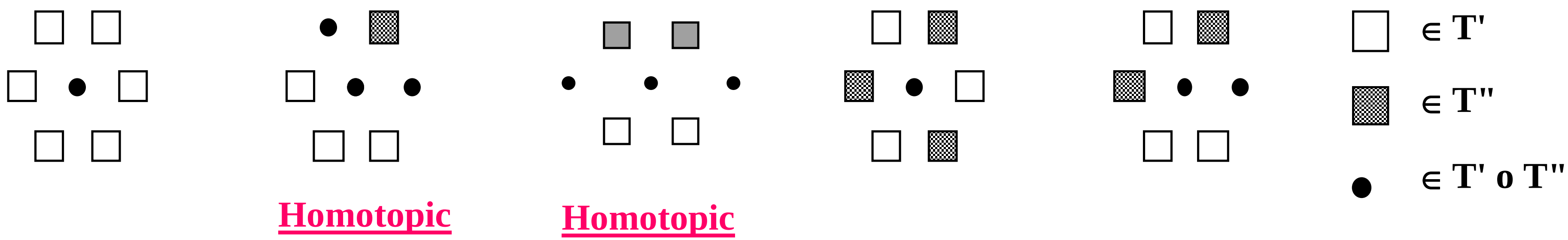
- Translation invariant
- rotation invariance of 60°
- Self-dual.



Homotopic Thinnings and Thickenings

A thinning or a thickening is homotopic if it relies on a double structuring element T = (T',T") which preserves homotopy.

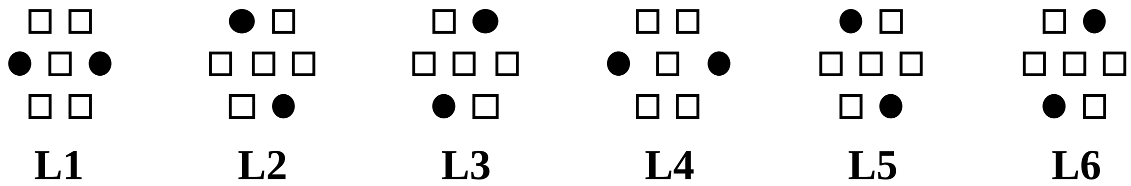
- ***Proposition*** *(J. Serra)* **:** In hexagonal grid, there exists only five Hit-or-Miss whose T’ and T" belong to the unit hexagon (all the others can be obtained by rotations, reflections and complementation):



and the change of the central point preserves homotopy if and only if the boundary of the hexagon involves only one change from "0" to "1". This property is fulfilled only for the elements of the second and of the third classes.

Sequential Thinning and Thickening

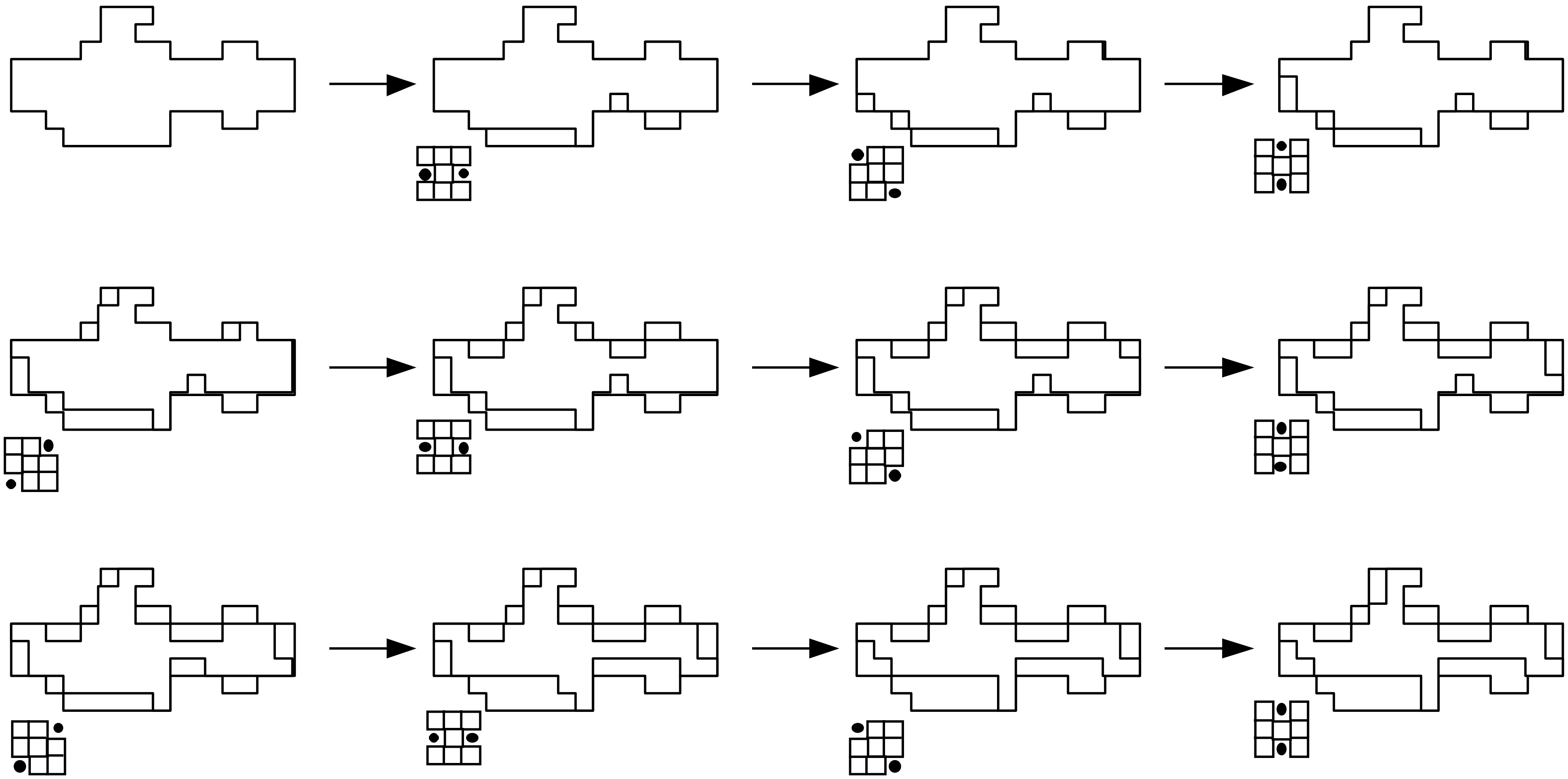
- In practice, "thinning" and "thickening" are used sequentially. For example, given a "structuring element" L=(L',L"), various "thinnings" are performed with all possible rotations of L and the transformation is repeated until idempotence:



$$\theta_L = \lim_{n\to\infty} (\theta_{L1} \ldots (\theta_{L5} (\theta_{L6})))^n$$

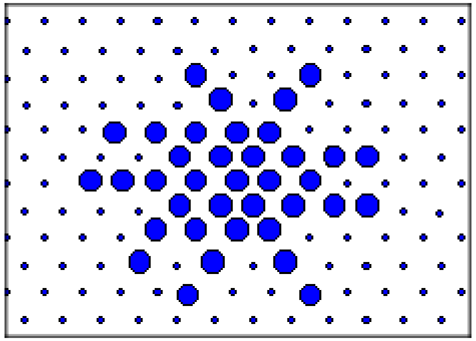
- For n large enough, the limit sequential thinning is ***anti-extensive, idempotent*** and ***preserves homotopy*** (but it is not increasing)

An Example of Sequential Thinning



Properties of the limit Thinning

- The result is not always thin. For example, the following set is not modified by "thinning" with the structuring elements L_i:



- The choice of the initial element and the order of the element sequence influences the final result.

- The thinnings are not very robust. Their lack of robustness is rendered by a number of parasite branches that depend on fine irregularities on the contours, and on the way the structuring element is rotated

Basic Structuring Elements (hexagonal grid)

Structuring element		Sequential Thinning	Sequential Thickening	Hit or miss
L	● □ □ □ □ □ ●	Skeleton of the shape	Skeleton of background	
M	● □ □ □ ● □ □	Skeleton of the shape with branches	Thickenning from isolated points	
D	● □ □ □ □ ● □	Homotopic marker	Quasi-convexe hull	
E	● ○ □ □ □ □ □	Pruning of skeleton	Pruning of background	Extreme points
F	□ □ □ □ □ □ □			Triple points
I	□ □ □ □ □ □ □			Isolated points

Homotopic (L, M, D, E)

Nonhomotopic (F, I)

References

On "Hit or Miss" :

- The "Hit or Miss" transformation can be considered as the starting point of Mathematical Morphology. It was introduced by J.Serra in 1965{SER65}, and, independently, by M.J.Golay in 1969 for hexagonal grids {GOL69}. However, the fundamental Banon-Barrera theorem is much more recent {BAN91}.

On Thinnings :

- Digital skeletonization by means of thinnings goes back to A.Rosenfeld who treated the square grid case {ROS70}. A systematic study of "thinning" in the discrete hexagonal cases is performed in {SER82,ch.11}, where one also finds for the first time the definition of homotopy for numerical functions (in ch.12), as it is presented here (there exist alternative definitions).