The birth of Mathematical Morphology

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June 1998

Foreword

It may be useful to add a few lines in preamble of the document below, in order to explain the context. Following G. Matheron's retirement, in december 1996, one could hear the most fanciful tales concerning the birth of Mathematical Morphology. I told G. Matheron about them, which prompted our decision to clarify this question publicly before the Research Committee of the Ecole des Mines. We sketched an outline together and I took charge of the first draft, which I transmitted to G. Matheron a few weeks later. We discussed it and produced the final version published here for the first time. We attended together the Research Committee meeting, during which we were granted a right of reply of a half hour. This was the last act of presence of G. Matheron at the Ecole des Mines de Paris. When somebody asked him which memories he had today of these remote days, he answered "They were the happiest in my life".

1 Introduction

L'action commence par conférer aux objets des caractères qu'ils ne possédaient pas par eux-mêmes, et l'expérience porte sur la liaison entre les caractères introduits par l'action dans l'objet (et non pas sur les propriétés antérieures de celui-ci).
Jean Piaget

At the time we begin to write this text, almost thirty years have passed, since that morning of April 1968, when we moved to the "Maintenon pavilion", on Saint-Honoré street, in Fontainebleau. As if to celebrate this anniversary, the famous Encyclopaedia of Mathematics (Reidel publisher) has opened its pages to Mathematical Morphology and has just established it as a section of mathematics .

Who were we ? What was our background ? How would the reflections and experiments that two men had pursued individually for four years, one in the friendly environment of IRSID (French Steel Institute), the other in the not so friendly environment of B.RG.M.(French Geological Survey), give a seed that would finally grow into a plant ?

These few pages will try to answer these questions. According to the common practice, we will use an impersonal presentation, and as the main characters of the story are only two, we will simply mention them by their initials.

2 Context

Before considering how Mathematical Morphology originated in 1964, it may be useful to describe briefly the professional backgrounds of its two founders at the beginning of that year, even though they were hardly aware of the theoretical turn they would take a few months later.

Georges Matheron was a mining engineer from the Corps des Mines, thirty-four years old, working with the B.R.G.M., in Paris, where he had his office, a secretary and an operator who translated his formulas into graphs or programs.

At the end of 1963, he had just achieved the doctrinal body of his theory concerning the estimation of deposits, called *Geostatistics*,and started in Algeria ten years before. It was obviously time for a break and for investigating new horizons beyond the mining world. His publication of reports even paused at that time, decreasing from fifty in ten years to none in ten months. He wouldn't return to geostatistics until January 1969, five years later, via universal kriging (except for two very short papers in april 1967).

The new topic that G. M. addressed for the first time, in October 1964, after ten months of silence, was the hydrodynamics of porous media. In a theoretical text of thirty-three pages, he attempted to link heat and hydraulic equations by probabilistic methods derived from Kolmogorov equations (G.M.55).

Jean Serra was a civil engineer from the Ecole des Mines de Nancy. He was twenty-four years old and was completing a bachelor's degree in philosophy which gave him the opportunity to discover Gestalt psychology as well as the genetic epistemology of Jean Piaget. One year before, he had been hired by IRSID as a PhD student in geostatistics in order to introduce this method for a detailed inventory of Lorraine iron orebody. This huge deposit, which had been the source of the economic prosperity in the region for more than a century, was on the verge of being outclassed by overseas ores, and its estimation was a priority.

Four different scales were to be considered. First of all, large profitable areas had to be delimited within the 600 $\mathrm{Km}^2$of the deposit, and the grades within

Figure 1: *Saint Pierremont mining face (gray bed). The migration of calcite produces nodules which hamper the estimation. Their analysis and modeling as random sets would be the subject of the second study carried out with the texture analyser [SER 68a] .*

them. But it also appeared that the hectometre scaled regionalisation of the panels to be kriged was generally affected by a strong *nugget effect* at the metric scale (fig 1). Local migrations of calcite introduced a bias in the drillings grades and the phenomenon had never been quantitatively studied. The last scale concerned petrographic arrangements (100 μ). For the enrichment of the ore, which had become essential, the grade of iron was less important than its petrographic distribution between oolites and chlorite cement (fig.2).

Therefore quantifying meant estimating the proportions of the petrographic phases and measuring their sizes.

In parallel with these economic objectives, J. S. also hoped to link the four scales through a point variogram, which would serve as a vital lead for this huge structural zoom and would cover the space from the micron to the hundred kilometer scale.

3 First chronology

May-June 1964 - Quantitative study of a thin section of the mine of la Mourière by J. S. The experimental device was just a slide projector. A thin section was the slide, and a centimetric square grid the screen. The petrographic phases found in each of the 6.000 grid vertices were read manually. Direct and cross variograms were calculated in the basic directions and diagonals, on a 1410 IBM computer.

The analysis of the results (summer'64) produced a great deal of information. Descriptors such as the hole effect, the co-regionalisation between phases, and the dissymmetry between them, occurred for the first time. But one goal was not achieved, since the variogram range did not give information which was precise enough on the three different phases (i.e. limonite oolites, chlorite cement and quartz grains).

October 1964 - Publication of G.M. 56 report, entitled "*Fonction aléatoire du type tout ou rien*", in which G. M. attempted to model the previous petrographic regionalisations in random terms, through the classical approach of the spatial probability distribution, *i.e.* based on a finite number of points. It was in this framework that he elaborated the Boolean model, formulated its covariance and more generally the moment based on n points.

January 1965 - Publication of J.S. 8 report, entitled "*contribution à l'analyse pétrographique quantitative*". This report, which detailed the results of petrographic surveys, introduced the concept of the structuring element, called "*spot*", and the hit or miss transformation. Two things occurred. First, J.S. could not link his variograms to the classical petrographic indices (roundness, circularity, packing and fabric) and second, the variograms themselves were only slightly granulometric. On the basis of these two observations, the author proposed starting from shapes which would be given *a priori* - the structuring elements - and to make them interact with the objects under study by means of the hit or miss transformation. According to their geometry (circle, straight line, pair of points), these "spots" would reveal different structural features of the objects (fig. 4). The report also mentioned the preparation of a "variograph" to analyse textures.

December 1964 - Publication of G.M.57 report, entitled "*Etude théorique des granulométries*" which chronologically followed J.S. 8, started from it, provided it with a formal framework, and, above all, extended it (fig 4 and 6). Documents J.S. 8 and G.M.57 can be considered as the very start of Mathematical Morphology. Report G.M.57 is composed of four parts :

a) Computation of the size distribution of spheres from their induced discs or their induced intercepts. The author found there an unexpected application of the grading formalism he had elaborated for geostatistics ;

b) Leaving the probabilistic framework, G. M. formalised the hit or miss transformation introduced in J.S. 6, limiting it to one-phase structuring elements. Calling it "*Serra's transformation*", G.M. described its algebraic properties and proposed an analytical representation ;

c) G. M. introduced the concepts of the opening and closing associated with erosion, defined their main algebraic properties, and interpreted openings according to balls of increasing radius in terms of granulometries ;

d) Returning to the probabilistic formalism, he expressed, by means of the notion of erosion, the moments of the Boolean model in a convenient framework, which forecast the theory of random sets.

Febrary 1965 - Publication of J.S. 9 report intitled "*La transformation en* *tout ou rien*" . It is a shortened version of J.S. 8 that takes G.M.57 results into account.

March 1965 - Publication of G.M. 58 report entitled "*Axiomatique des milieux poreux aléatoires*". This text established a series of additional results of the hit or miss transformation, concerning topological properties and convexity. The second part, more probabilistic, outlined the theory of random sets.

May 1965 - Publication of J.S. 13 report, "*L'analyse des textures par la géométrie aléatoire* ", written in preparation of the patent of July 2, 1965. The PhD student was bold enough to write : "nowhere, in petrography or anywhere else, is it possible to say that a material object *has* a shape". J. Piaget and the Gestalt Theorie were not far...

July 1965 - Registration of the first patent of the texture analyser, inventor : J. S., company : IRSID. One can see in figure 3 the front page of the corresponding U.S. deposit.

The registration of this patent on July 2, 1965, ended the first year of Mathematical Morphology. The G.M. 57 report was the basis for the morphological part of G. M.'s book [3] ; J.S. 6 report was published in the "bullein du BRGM in 1966 [10] J.S. 13 report was presented on November 2, 1965, by its author, to the scientific commitee of IRSID.[9]. Finally, J.C. Klein was hired on January 1st 1965 to build the electronics of the texture analyser.

4 Second chronology

Surprisingly, the first year, which was so rich in fruitful exchanges, was followed by two years of almost complete silence on the subject. G. M. turned to hydrodynamics (eight studies written during that period) and the only, more or less, morphological text that he wrote, G.M. 61 report, bore upon permeabilities (laws of the first point of contact, model of independent channels). The main reason for this situation was the development of the texture analyser that took the entire year of 1965

From January 1966, the machine allowed the construction of every possible structuring element based on a straight line, and a few months later, those of the second dimension was still limited, it was a small technical revolution, whose success and performances were greatly to the credit of J.C. Klein.

Whereas the first, manual, study on the thin section from La Mourière had cost three months' work, only ten minutes were needed for a sampling ten times as dense. The idea to use the object under study for its own memory changed completely the access to experimentation, and without this novel means, it is likely that Mathematical Morphology would never have developed (this is also

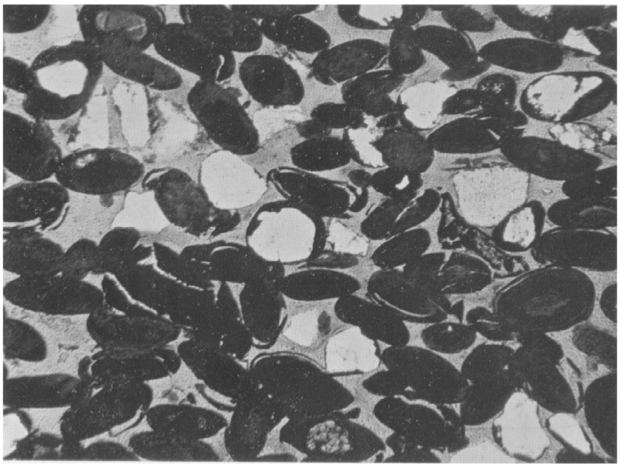
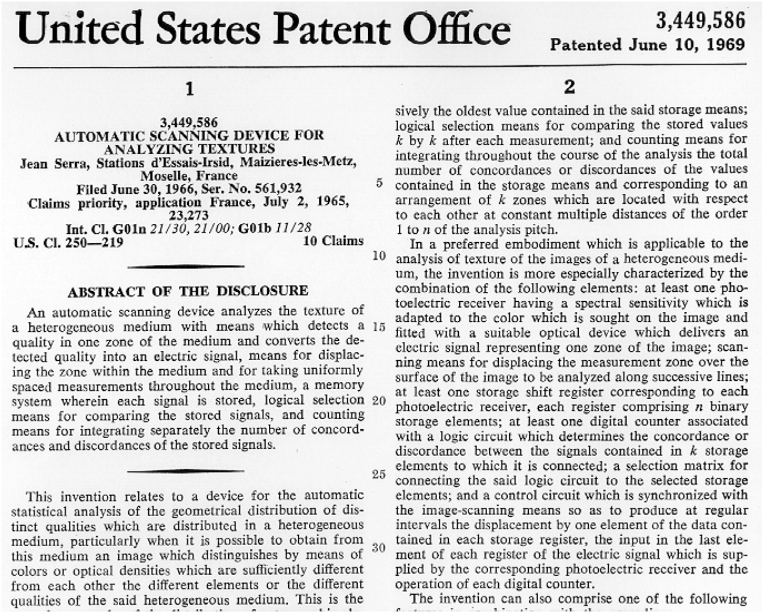


Figure 2: *Detail of a thin section from the mine of La Mourière (red bed), on which the first experimental, and manual, study of Mathematical Morphology was focused, and the first use of the texture analyser [SER 66]. Limonite oolites, dark and sometimes formed around a quartz germ, are cemented by chlorite.*

the reason why it blossomed in France rather that in the United States : a lot of American scientists are too proud of their computers to try to bypass them).

The whole year of 1966 was devoted to the inventory of "good" structuring elements and to establishing logical relationships between them (e.g. : to derive a linear erosion curve comes down to have the structuring elements of these erosions preceded by a couple of points 01). The research on calcite migration and chloritization resumed. The latter only needed a series of thin sections which were easily obtained, the former led to a photographic survey of mining faces from St-Pierremont (fig.1). The migration of calcite was dense enough, hence visible, in that part of the deposit to try to exploit directly some thirty slides with the texture analyser. And it worked (J.S. 17) !

The silence of texts did not reflect any divergence of interests between the two men. On the contrary, during these two years they met every week, from October to January, after the course of probability that G. M. gave at the Ecole des Mines de Nancy, assisted by J. S. During this hour ritually spent in a local pub, they exchanged information, they invented the name "*Mathematical Morphology*" for the new discipline, as well as other terms, and they gradually worked out a paper surveying the whole structural information that could be obtained through a scanning of one or two parallel straight lines. Since J.S. left for sixteen months at the beginning of December to do his national service, G.M. took care of the final writing of the text, intended to be a kind of manifesto of Mathematical Morphology. IRSID hired A. Haas to replace J.S.. Haas played a



United States Patent Office

3,449,586
Patented June 10, 1969

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3,449,586
AUTOMATIC SCANNING DEVICE FOR ANALYZING TEXTURES
Jean Serra, Stations d'Essais-Irsid, Maizieres-les-Metz, Moselle, France
Filed June 30, 1966, Ser. No. 561,932
Claims priority, application France, July 2, 1965, 23,273
Int. Cl. G01n *21/30, 21/00;* G01b *11/28*
U.S. Cl. 250—219 10 Claims

ABSTRACT OF THE DISCLOSURE

An automatic scanning device analyzes the texture of a heterogeneous medium with means which detects a quality in one zone of the medium and converts the detected quality into an electric signal, means for displacing the zone within the medium and for taking uniformly spaced measurements throughout the medium, a memory system wherein each signal is stored, logical selection means for comparing the stored signals, and counting means for integrating separately the number of concordances and discordances of the stored signals.

This invention relates to a device for the automatic statistical analysis of the geometrical distribution of distinct qualities which are distributed in a heterogeneous medium, particularly when it is possible to obtain from this medium an image which distinguishes by means of colors or optical densities which are sufficiently different from each other the different elements or the different qualities of the said heterogeneous medium. This is the

2

sively the oldest value contained in the said storage means; logical selection means for comparing the stored values *k* by *k* after each measurement; and counting means for integrating throughout the course of the analysis the total number of concordances or discordances of the values contained in the storage means and corresponding to an arrangement of *k* zones which are located with respect to each other at constant multiple distances of the order 1 to *n* of the analysis pitch.

In a preferred embodiment which is applicable to the analysis of texture of the images of a heterogeneous medium, the invention is more especially characterized by the combination of the following elements: at least one photoelectric receiver having a spectral sensitivity which is adapted to the color which is sought on the image and fitted with a suitable optical device which delivers an electric signal representing one zone of the image; scanning means for displacing the measurement zone over the surface of the image to be analyzed along successive lines; at least one storage shift register corresponding to each photoelectric receiver, each register comprising *n* binary storage elements; at least one digital counter associated with a logic circuit which determines the concordance or discordance between the signals contained in *k* storage elements to which it is connected; a selection matrix for connecting the said logic circuit to the selected storage elements; and a control circuit which is synchronized with the image-scanning means so as to produce at regular intervals the displacement by one element of the data contained in each storage register, the input in the last element of each register of the electric signal which is supplied by the corresponding photoelectric receiver and the operation of each digital counter.

The invention can also comprise one of the following

Figure 3: *Fac simile of the begining of the first U.S. patent of the Texture Analyser*

significant role in the experimental part of the manifesto, which was published in two successive papers in the *Annales des Mines* in December 1967 [HAA 67].

Looking back on the year 1967, it appears that G.M. and J.S. were both busy finishing previous work, and preparing the creation of C.M.M., even if they did not suspect it yet. The national service as a first lieutenent in a military research center allowing some free time, J. S. concluded his mining studies, during his military service, with a PhD thesis in engineering (October 1967), and his works in mathematical geology by writing the two papers J.S. 17 and J.S.18 1968 , one on the metric scale, the other on the change of scale from the micron to the kilometer (publications [12] and [13] respectively). For their author, everything had been said on the descriptive potential of the structuring element consisting of a "couple of points". At the same time, he was preparing a new patent for the texture analyser, to be later registered and exploited for the next twenty years with profit by ARMINES.

Still in 1967, G. M., after he had chosen to take some distance from Geostatistics, left B.R.G.M. and wrote the first important theoretical text on Mathematical Morphology (G.M. 74) which prefigured the "*Theory des ensembles aléatoires*" [4] , and the first chapters of "*Random sets and Integral Geometry* " [7] . Finally, he gathered his morphological papers of the first period

(1964-1965) and some texts on hydrodynamics (1965-1966) in a book entitled "*Eléments pour une théorie des milieux poreux*" [3] .

On April 20, 1968, G. M. moved to the Maintenon pavilion, in Fontainebleau, renamed Centre de Morphologie Mathématique. The Ecole des Mines de Paris created the "Centre de Morphologie Mathématique", and appointed him director. Despite the imposing size of the building, the center was reduced to two people: G.M. and J. S. . The later having completed his military service in April, had just been hired as "Maître de Recherches" by the Ecole des Mines, and appointed to C.M.M..

We know now, thirty years later, that once transplanted into the ground of the Ecole des Mines, the burgeoning plant would grow to the point that one can hardly find today an image processing program that does not involves, more or less, Mathematical Morphology.

Chronology sharpens the events, makes more precise their apparitions, but says nothing about their reasons. What happened during the three years we have just skipped through, and above all during the first one ? What kind of impulse did these two men give by working together with practically no intellectual connection with the people around them ? Why did this methodology gradually gain in influence instead of another ? Is it due to the mathematical expression of its concepts or to a better ability to render the concepts of the time, and integrate them into a unified approach ? Or is it simply the result of the technical evolution of computer science that enabled the synchronized development of Mathematical Morphology as a sort of by-product ?

5 Are the essentials of a scientific theory contained in its mathematical corpus ?

First remark : With the patent and the development of the texture analyser, four papers marked the first year of morphology, namely (G.M.57) (J.S.8) (G.M.58) and (J.S.13). However, the two texts by J. S. are mathematically thin whereas the two studies by G. M. demonstrated an impressive technical virtuosity, and an even greater originality of expression : new concepts demand new formalisms.

Second remark : In (G.M.57), G. M. rediscovered the set addition and subtraction defined by H. Minkowski (1903). These operations had been studied in detail in a brilliant work by H. Hadwiger which had been published seven years before [2] . This book even mentioned for the first time the opening and closing, and their extensive property. Hadwiger's approach belonged to a school of thought called Integral Geometry, which has from Minkowski to this day, constructed set models and tried to describe them with appropriate *numbers*. In parallel, since the early sixties, practitioners inspired by those models for quantifying purposes have founded the International Society of Stereology.

Though never in ninety-five years of theory, nor in thirty-five years of practice, the idea of using a set as a probing tool to analyse another (J.S. 13) or to measure its size distribution (G.M.57) occurred in these groups. And what is more, having realized how closely related were Mathematical Morphology and Stereology, G. M. and J. S. presented their ideas before the Society of Stereology, in 1971 [5] and [14] , where they were received with rather mixed feelings. The postulate that one can study a set *only* by associating it with numbers was so deeply rooted in their minds, and so implicitely, that the stereologists were just unable to imagine other ways of thinking. The *physical* intuition of morphology that perceiving a set, or an image, means to transform it, either qualitatively or quantitatively, was too far removed from theirs.

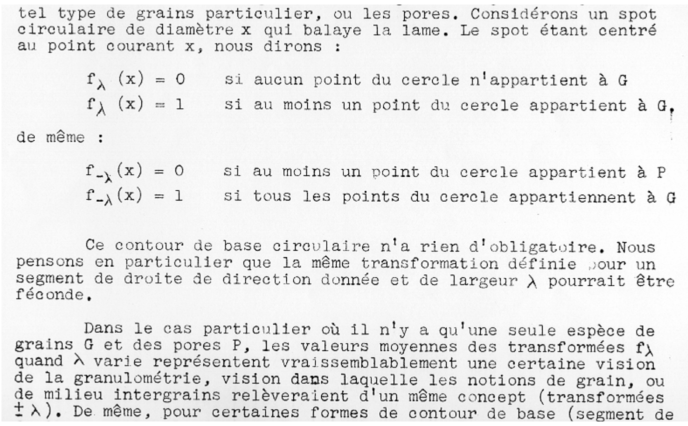
And yet the new approach used to open number of questions. For example, in Minkowski addition, the two operand sets play a symmetrical part. Then how could somebody give credit to that approach which was based both on Minkowski sum and on the dissymmetric parts it assigned to the two sets ? Here is another example: according to the theory, dilation was to be a more robust operation than erosion, although no experiment could verify the fact. Also, one could wonder whether such a theory would apply to gray level images, or uniquely to binary ones. Etc..But the stereologists did not pay attention to theses aspects.

Thus, the first germs of the new method, as well as the first opposition to it, brought to the fore an intuitive level deeper than that of mathematics. The latter appears as a second layer, or as an interface that transmutes the basic intuitions into formal models for future experiments. The one who would limit the epistemology of Mathemetical Morphology to set models would make the same type of mistake as to reducing the first theory of relativity to Lorentz transformation .

6 Are ideas just the fruits of technological evolution ?

Up to what point can we consider as ours those theoretical inventions that we have indulged ourselves in thinking they were original, but whose succession in time seems surprisingly correlated with the development of computer technology?

From this point of view, the evolution of Mathematical Morphology over the past thirty years is instructive. Two-dimensional operators were discovered four years before television technology enabled their implementation (1969); the memory of the size of an image, with its very peculiar loop functioning, occurred in 1973, simultaneously with a growing interest for binary iterative algorithms (ultimate erosion, skeleton, thinning). Five years later, the numerical memory opened the door to gray tone image processing, precisely at the


tel type de grains particulier, ou les pores. Considérons un spot circulaire de diamètre x qui balaye la lame. Le spot étant centré au point courant x, nous dirons :

$f_\lambda(x) = 0$ si aucun point du cercle n'appartient à G
$f_\lambda(x) = 1$ si au moins un point du cercle appartient à G,

de même :

$f_{-\lambda}(x) = 0$ si au moins un point du cercle appartient à P
$f_{-\lambda}(x) = 1$ si tous les points du cercle appartiennent à G

Ce contour de base circulaire n'a rien d'obligatoire. Nous pensons en particulier que la même transformation définie pour un segment de droite de direction donnée et de largeur λ pourrait être féconde.

Dans le cas particulier où il n'y a qu'une seule espèce de grains G et des pores P, les valeurs moyennes des transformées f_λ quand λ varie représentent vraissemblablement une certaine vision de la granulométrie, vision dans laquelle les notions de grain, ou de milieu intergrains relèveraient d'un même concept (transformées $\pm\lambda$). De même, pour certaines formes de contour de base (segment de

Figure 4: *Extract of J.S.8 paper : original version of dilation and erosion (the combined hit or miss transformation appeared for the first time in J.S.13).*

time when morphological filtering and the watershed algorithm were invented. More recently, the morphological theory for connection, that governs automatic segmentation of images and sequences, was developed during the nineties, just when the multimedia applications arose. What a lot of coincidences !

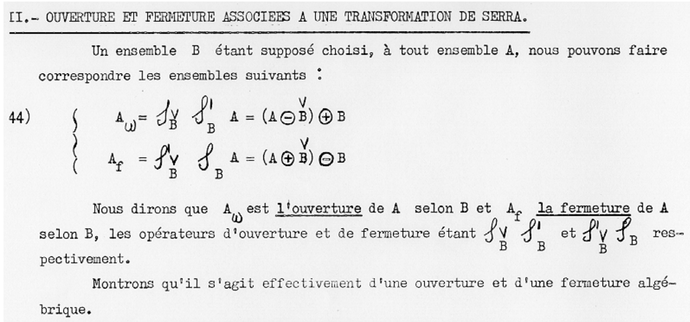
Even if we are not always aware of it, we all "surf" on successive technological waves. However, this this subconscious pressure applies uniformly over all the 1500 image processing laboratories in the world. Why then strong ideas do crystallize in a few places only, and at some privileged periods ?

7 How an idea was born

To all these paradoxes, we must add this one : why say that in 1964 two people invented Mathematical Morphology whereas at that time, they knew nothing about skeletons, levelings, connectivity on lattices, not even lattices ? The use of undetermined verbal forms sometimes look improper...

Here, the ambiguity of the paradox comes from the fact that a *process* is confused with its *results*. In 1964, the body of results was of course limited, and did not exactly forecast those which would follow. But the way of thinking that came with them, which was created at that time, turned out to be an original schooling in understanding image analysis. To illustrate this point, the birth of the concept of a granulometry deserves some attention.

The first question was : how to know the size of oolites and of chlorite cement in Lorraine iron ores (at least one mean volume parameter, and more if



II.- OUVERTURE ET FERMETURE ASSOCIEES A UNE TRANSFORMATION DE SERRA.

Un ensemble B étant supposé choisi, à tout ensemble A, nous pouvons faire correspondre les ensembles suivants :

44) $$\begin{cases} A_\omega = \mathcal{S}_{\check{B}} \, \mathcal{S}'_B \, A = (A \ominus \check{B}) \oplus B \\ A_f = \mathcal{S}'_{\check{B}} \, \mathcal{S}_B \, A = (A \oplus \check{B}) \ominus B \end{cases}$$

Nous dirons que A_ω est l'ouverture de A selon B et A_f la fermeture de A selon B, les opérateurs d'ouverture et de fermeture étant $\mathcal{S}_{\check{B}} \mathcal{S}'_B$ et $\mathcal{S}'_{\check{B}} \mathcal{S}_B$ respectivement.

Montrons qu'il s'agit effectivement d'une ouverture et d'une fermeture algébrique.

Figure 5: *Extract of G.M. 57 paper: First occurrence of the opening and closing associated with an erosion and its related dilation.*

possible) ?

First answer : The results of the first point variograms led J.S. to the following proposition (fig. 4)J.S. was wrong when considering dilations or distance functions as granulometric tools, but he did feel the need for convex structuring elements. He formulated the question differently in terms of "*sets* → *sets*" transformations (and not "*sets* → *numbers*") which was new, and introduced the paradigm for this type of transformations.

Second step : G.M. revived the idea of transforming sets and showed that dilations using disks should be combined with their dual operations to obtain a granulometric result (fig.5 and 6).

Once again a good insight emerged out of a questionable mathematical statement : the function $S(\lambda)$ was certainly not decreasing, but the same was true already for dilations according to disks. The strength of G.M. approach actually came from the introduction of *idempotent* operators.Indeed, this property was proved for opening a few pages before, but its author did not venture yet to substitute it (with two other axioms) as *constitutive* of the physical concept of granulometry. *Third step* : Since the circular opening was still out of reach of the technical tools available at that time (texture analyser of 1965), G.M. and J.S. turned to the linear erosion, which was experimentally accessible.In 1966, a recently engaged engineer called André HAAS joined the two morphologists. Together, they systematically investigated all usefull structuring elements supported by one line or by two close parallel ones. From linear erosions, they derived measurements of the mean volume, such as the star for instance, which brought an answer to the question at hand [1].

Epilogue : Eight years later, G.M. finally came up with the morphological concept of granulometry under its complete and final form [MAT 72b], based

Plaçons-nous dans le cas où l'ensemble A est mesurable, et soit $S = S(o)$ sa mesure. Désignons par :

$S(\lambda)$ la mesure de A_{f_λ} (fermeture de A selon B_λ)

$S(-\lambda)$ la mesure de A_{ω_λ} (ouverture de A selon B_λ).

$S(\lambda)$ est <u>une fonction non décroissante de λ.</u>

En effet, soit $0 < \lambda \leq \mu$. D'après (50), B_μ est ouvert selon B_λ, et, d'après la conséquence 4, on a les inclusions :

$$A_{f_\lambda} \subset A_{f_\mu}$$

$$A_{\omega_\lambda} \supset A_{\omega_\mu}$$

Par suite, on a aussi :

$$\begin{cases} S(\lambda) \leq S(\mu) \\ S(-\lambda) \geq S(-\mu) \end{cases}$$

Cette propriété montre que $S(\lambda)$ possède les caractéristiques d'une courbe granulométrique cumulée. Elle en possède aussi la signification physique, comme nous allons le voir, et c'est pourquoi nous lui donnerons le nom de <u>courbe granulométrique généralisée</u> de la régionalisation en tout ou rien A.

Figure 6: *Extract of G.M. 57 paper : First version of the granulometry (B_λ being the open ball of diameter λ)*

on a series of set axioms. Meantime, the orebody of Lorraine had lost any all its significance for the French iron and steel industry...

Thus, ideas first need time to take shape, then to expand and link together. Platonists are not always aware of this point. Those who try to construct a mathematical structure are sometimes lured by the strong impression that "*it does exist*" somewhere, outside the sensible world. But because they interpret this impression from a retrospective, therefore static, point of view, they tend to forget that they feel that "*it does exist*" because "*it does resist*" (a feeling that is not so Platonic..)

In our story, experimental facts are never used to verify ready-made formulas. They appear on the contrary as a support to help an idea under construction to find its structure and shape.

And the dialog that takes place between the two researchers via their texts fulfils exactly the same function : it is through successive exchanges and modifications that ideas progressively take shape. Moreover it does not seem necessary, nor even desirable, that the two protagonists be too much alike, that they have the same mathematical background for instance. Their dialog is successful because they view reality from two different angles, but within the same frame of thought.

8 Conclusion

To conclude, we will come back to the paradox of the initial invention. From 1968, some previous results were to be developed (Boolean model, hit or miss transformation), others not (star, radii of curvature), and the original machine evolved rapidly to other proptotypes and commercial devices (Leitz, Allen Bradeley, etc). But the essential point was elsewhere and was based on the choice of specific methodological and experimental lines, which was already effective in 1968. A style was born, with its presuppositions, its strenghts and its weaknesses, which was to influence the researchers almost without their knowing, so that today one immediately recognizes, after reading a few paragraphs, whether or not a scientific paper comes from the school of Mathematical Morphology .

As regards team work, it also established a special type of relationships. Subsequently, creations like skeletonization, morphological filtering, or levelings (to mention only a few) would follow the same modes of exchange, between G.M. and other researchers, the same rythms, even the same duration, as those which its two founders had established intuitively from the very first years of Mathematical Morphology.

9 Appendix

Translation of figure 4 : *Let us consider a circular spot of diameter x scanning the section. The spot is centered in the current point x, we shall say :*

$$f_\lambda(x) = 0 \quad \text{if no point of the circle belongs to } G$$
$$f_\lambda(x) = 1 \quad \text{if at least one point of the circle belongs to } G$$

Similarly

$$f_{-\lambda}(x) = 0 \quad \text{if no point of the circle belongs to } G$$
$$f_{-\lambda}(x) = 1 \quad \text{if at least one point of the circle belongs to } G$$

It is not necessary to have a circular contour. For example, using the same transformation based on a straight line segment of width λ, in a given direction, could be fruitful.

In the particular case where there is only one sort of grains G and one sort of pores P, the mean values of the transformed sets f_λ when λ varies, probably represent a certain interpretation of the granulometry, according to which the notion of grains, or intergrain media would pertain to the same concept ($\pm\lambda$ transformation).

Translation of fig 5 : *Opening and closing associated with Serra's transformation*

Given a set B, to any set A can correspond the following sets :

$$\begin{cases} A_w = \mathcal{S}_{\check{B}}\mathcal{S}'_B \quad A = (A \ominus \check{B}) \oplus B \\ A_f = \mathcal{S}'_{\check{B}}\mathcal{S}_B \quad A = (A \oplus \check{B}) \ominus B \end{cases}$$

We say that A_w is the opening of A according to B and A_f the closing of A according to B, the opening and closing operators are $S_{\check{B}}S'_B$ and $S'_{\check{B}}S_B$ respectively.

Let us show that these transformations are indeed algebraic opening and closing.

Translation of fig 6 : *Consider the case where the set A is measurable, and let $S = S(0)$ be its measure. We denote*

$$\begin{array}{lll} S(\lambda) & \text{measure of } A_{f_\lambda} & (\text{closing of } A \text{ according to } B_\lambda) \\ S(-\lambda) & \text{measure of } A_{w_\lambda} & (\text{opening of } A \text{ according to } B_\lambda). \end{array}$$

$S(\lambda)$ is a non decreasing function of λ. Indeed, let $0 < \lambda \leq \mu$. From relation (50) it follows that B_μ is an opened set according to B_λ, and, from implication 4, we have the inclusions :

$$\begin{array}{l} A_{f_\lambda} \subset A_{f_\mu} \\ A_{w_\lambda} \supset A_{w_\mu} \end{array}$$

Hence, we have also :

$$\begin{cases} S(\lambda) & \leq & S(\mu) \\ S(-\lambda) & \geq & S(-\mu) \end{cases}$$

This property shows that $S(\lambda)$ has the characteristics of a cumulated granulometric curve. It also has its physical meaning, as we shall see, this is the reason why we call it a generalized granulometric curve of the hit or miss regionalization A.

References

The documents mentioned in the first list below only concern works done between 1964 and 1967. Those which were published later are indicated in the second list. All these texts are available from the library of the Ecole des Mines de Paris, or from the libraries of the Centre de Morphologie Mathématique (CMM), and the Centre de Géostatistique (CG).

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